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REGULAR ARTICLE

Evaluation of translational friction coefficients of micro-sized spherical probes in nematic liquid crystals

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Abstract We study the translational friction coefficients of a spherical micrometric probe moving in nematic liquid crystalline fluids, by solving numerically the constitutive hydrodynamic equations of uncompressible isothermal nematic fluids (Leslie–Ericksen equations). The nematic medium is described by a vector field, which specifies the director orientation at each point and by the velocity vector field. Simulations of director dynamics surrounding the moving probe are presented, and the dependence of translational diffusion upon liquid crystal viscoelastic parameters is discussed. The time evolution of director field is studied in the presence of an orienting magnetic field in two characteristic situations, i.e. direction of motion parallel and perpendicular to field. In particular, a detailed analysis is given for the case of a spherical probe in rectilinear motion in nematic MBBA (4-methoxibenzylidene-4′-*n*-butylaniline), together with a comparison with other nematogens.

Keywords Nematic liquid crystals · Leslie-Ericksen · Translational friction coefficients

1 Introduction

In this study we present a systematic analysis of the translational friction acting on micro-sized probes moving in nematic fluids, by solving numerically the constitutive hydrodynamic equations of nematics under some approximations.

Contribution to the Nino Russo Special Issue.

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In the past, several hydrodynamic and kinetic interpretations of experimental data on translation friction (or diffusion) coefficients of solutes in nematic solvents have been derived. Diogo [1] found analytical expressions for the friction acting on a spherical molecule in a nematic in the presence of an external field, under severe approximations. Franklin [2] used a modification of Kirkwood theory to relate translation parallel and perpendicular diffusion coefficients of probes in nematic to viscoelastic parameters, order parameter and molecular shape, deriving also an expression for rotation diffusion coefficients, and lately [3] employed hydrodynamic theory to interpret experimental findings of diffusion coefficients of molecular probes. Khare et al. [4] presented a kinetic treatment to study translation diffusion in nematic fluids, and compared their results with computer simulations.

Experimental measures of translation diffusion coefficients of molecular probes in liquid crystals have been also obtained using several techniques. Yun and Fredickson [5] measured translation diffusion of molecules tagged with ^{14}C radioactive isotopes by liquid scintillation counting in various liquid crystals, determining the parallel and perpendicular diffusion with respect to an applied magnetic field. Moseley and Lowenstein [6,7] studied the diffusive motion of methane and chloroform molecules in liquid crystals. More generally Krüger [8] discussed several experimental techniques like NMR, MTR and QENS to measure diffusion coefficients in nematic and smectic phases, together with their theoretical interpretation. Recently, Spiegel et al. [9] employed forced Rayleigh scattering to study the diffusion of methyl-red in 5CB (4′-n-Pentyl-4-cyanobiphenyl).

In this work, we shall describe numerically the translation of a spherical probe in nematic liquid crystals, solving the constitutive hydrodynamic equations of nematic fluids under the standard hypothesis that the velocity field is the sum of isotropic fluids. In particular, we shall present a qualitative analysis of the director field patterns created by the perturbation caused by the moving probe. The paper is organised as follows. Basic methodology is presented in Sect. 2, first by reviewing the translational behaviour of a probe in rectilinear motion in an isotropic Newtonian fluid in condition of creeping flow, and then by generalising the same methodology to nematic fluids. The computational approach is also described. Section 3 is devoted to the analysis of several numerical and asymptotic results. Translation coefficients are evaluated for the case of spherical probes in the presence of external magnetic fields, in different low-molecular weight nematogens. Our findings are summarised in Sect. 4.

2 The model

2.1 Isotropic fluid

It is convenient to first summarise the evaluation of translational friction coefficients for a moving probe in an *isotropic* isothermal incompressible fluid under conditions of creeping flow [10]. In their most general form Navier–Stokes (NS) equations for an isotropic isothermal incompressible fluid describe the time evolution of fields $v_j(\mathbf{r}, t)$ which are components along the fixed axes of a laboratory frame $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ of the velocity field vector $\mathbf{v}(\mathbf{r}, t)$ in the space point $\mathbf{r}$ of Cartesian coordinates r_1, r_2, r_3 or polar coordinates r, θ, ϕ at time t.

In the absence of external forces, NS equations read

$$\rho \frac{\mathrm{d}v_j}{\mathrm{d}t} = -\frac{\partial p}{\partial r_k} + \mu \nabla^2 v_j \tag{1}$$

where $p(\mathbf{r}, t)$ is the pressure, while μ is the dynamic viscosity; $\mathrm{d}/\mathrm{d}t$ is the material time derivative $\mathrm{d}/\mathrm{d}t = \partial/\partial t + v_k \times \partial/\partial r_k$. Einstein's convention holds here and everywhere else in this paper unless otherwise stated.

Let us now consider a fluid moving around a fixed spherical probe of radius R, with centre in the laboratory frame origin, at a constant velocity along a chosen axis, e.g. $\mathbf{e}_3$. This is tantamount to consider a sphere moving at constant velocity $-V$ along the $\mathbf{e}_3$ axis, with the fluid having the same velocity on the probe surface, and zero flow or velocity at $r \to \infty$ [11], and thus one needs to solve Eq. (1) for the boundary conditions $v_j = 0$ for $r = 0$, assuming stick conditions, and $v_j = V\delta_{j,3}$ for $r \to \infty$. In conditions of creeping flow, or zero material derivative, the solution is analytic [11]

$$p = -\frac{3}{2}\mu V \frac{R r_3}{r^3} \tag{2}$$

$$\frac{v_j}{V} = \left[1 - \frac{R}{4r}\left(\frac{R^2}{r^2} + 3\right)\right]\delta_{j,3} + \frac{3 R r_j r_3}{4r^3}\left(\frac{R^2}{r^2} - 1\right). \tag{3}$$

The force acting on the sphere can now be calculated as

$$F_i = -\int_S \mathrm{d}S \sigma_{ij} m_j \tag{4}$$

where the integral is extended to the sphere surface and m_j is the jth component in each surface point of the unitary normal vector $\mathbf{m}$; σ_{ij} is the ij component of the stress tensor σ, for a Newtonian fluid defined as

$$\sigma_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial v_i}{\partial r_j} + \frac{\partial v_j}{\partial r_i}\right). \tag{5}$$

Integration of Eq. (4) is easily accomplished to give the only non-zero component along the axis of motion [10] $F_3 = 6\pi\mu R$, i.e. Stokes' law for the translation friction coefficient is found

$$\xi = 6\pi\mu R. \tag{6}$$

2.2 Leslie–Ericksen equations

Let us now review briefly the basic features of the hydrodynamic description of nematic incompressible isothermal fluids. A nematic fluid is described by the fields $\mathbf{n}(\mathbf{r}, t)$, a unitary vector which specifies the director orientation in each point at a given time and $\mathbf{v}(\mathbf{r}, t)$, i.e. velocity and pressure. Constitutive Leslie–Ericksen equations describe the time evolution of the vector components n_j, v_j and of pressure p in the following form (neglecting so-called inertial components in the director equation) [12]:

$$\rho \frac{\mathrm{d}v_j}{\mathrm{d}t} = \frac{\partial \sigma_{ij}}{\partial r_j} \tag{7}$$

$$G_j + g_j + \frac{\partial \pi_{ij}}{\partial r_j} = 0 \tag{8}$$

where σ_{ij} is the stress tensor, ρ is the density of the bulk, G_j is the jth component of a generic external force (typically generated by an external electric or magnetic field), g_j is an internal force that act on the director, derived from elastic and viscous contribution (see below), and π_{ij} is a tensor coming from purely elastic effects:

$$\pi_{ij} = \frac{\partial W}{\partial n_{j,i}} \tag{9}$$

where $n_{j,i}$ is the shorthand form for $\partial n_j/\partial r_i$, and W is the nematic elastic energy

$$W = \frac{1}{2}K_{11}(\nabla \cdot \mathbf{n})^2 + \frac{1}{2}K_{22}(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + \frac{1}{2}K_{33}(\mathbf{n} \times \nabla \times \mathbf{n})^2 \tag{10}$$

where K_{11}, K_{22} and K_{33} are elastic constants. Terms dependent upon viscous properties are the stress tensor components

$$\sigma_{ij} = -p\delta_{ij} - \pi_{ik}\frac{\partial n_k}{\partial r_j} + \sigma'_{ij} \tag{11}$$

here σ'_{ij} is the contribution to the stress tensor coming from purely viscous effects

$$\sigma'_{ij} = \alpha_1 n_k n_p A_{kp} n_i n_j + \alpha_2 n_i N_j + \alpha_3 n_j N_i + \alpha_4 A_{ij} + \alpha_5 n_i n_k A_{kj} + \alpha_6 n_j n_k A_{ki}. \tag{12}$$

In Eq. (12), α_i are the well-known Leslie's coefficients which characterize, together with elastic constants, the nematic fluid under investigation; matrix A_{ij} is simply $A_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial r_j} + \frac{\partial v_j}{\partial r_i}\right)$ and vector N_i is dependent on the director and velocity components, i.e. $N_i = \frac{\mathrm{d}n_i}{\mathrm{d}t} - \omega_{ij}n_j$, where ω_{ij} is $\omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial r_j} - \frac{\partial v_j}{\partial r_i}\right)$. Finally the internal force g is given by

$$g_i = \lambda_L n_i - \frac{\partial W}{\partial n_i} - \gamma_1 N_i - \gamma_2 A_{ij} n_j \tag{13}$$

where $\gamma_1 = \alpha_3 - \alpha_2, \gamma_2 = \alpha_3 + \alpha_2$ and λ_L is a Lagrange multiplier related to the constraint on the norm of the director $n_i n_i = 1$. LE equation are reduced to NS equation by neglecting all viscous coefficients except α_4, provided that μ is identified with $\alpha_4/2$.

Computational solutions of LE equations are difficult, due to their intrinsical non-linear character. In the present case, one should consider the simultaneous solution of Eqs. (7) and (8), in order to account exactly for backflow effects due to the director re-orientation on the fluid velocity, i.e. on the time evolution of the velocity field. It is, however, reasonable, to a first approximation, at least for the case of creeping flow, to consider the velocity field as a stationary and Newtonian quantity. Exact numerical calculations for the interpretation of magneto-rheological experiments [13–15] have shown that in several cases the velocity field can indeed be approximated by the stationary solution of NS equations. We shall therefore assume that components v_k entering director equations are *known functions* given by Eq. (3) for a sphere. Writing explicitly the director equations one gets

$$\frac{\partial n_i}{\partial t} = \frac{\lambda_L n_i}{\gamma_1} + \omega_{ik}n_k - \frac{\gamma_2}{\gamma_1}A_{ik}n_k - v_k\frac{\partial}{\partial r_k}n_i + \frac{K}{\gamma_1}\frac{\partial^2}{\partial r_k^2}n_i + \frac{\Delta\chi B_i B_k}{\mu_0\gamma_1}n_k \tag{14}$$

where the additional spherical approximation of elastic energy $K_1 = K_2 = K_3 = K$ has been assumed for simplicity and $\Delta\chi$ is the magnetic anisotropy. The external force has been explicitly written as the result of a magnetic field $\mathbf{B}$ coupled with the director [16]. Equation (14) is only subject to the unitary constraint on the director components and to boundary and initial conditions.

2.3 Computational methodology

Our strategy is then the following: first we solve numerically for the director components in time and space, then we substitute in Eq. (4) to calculate the force acting on the probe and the friction coefficient. We start by scaling Eq. (14), by introducing a convenient set of scaled quantities $v_i^* = v_i/V$, $r_i^* = r_i/R$ and $t^* = Vt/R$. Scaled director equations are then

$$\frac{\partial n_i}{\partial t^*} = \lambda n_i + \omega^*_{ik}n_k - \gamma A^*_{ik}n_k - v^*_k\frac{\partial}{\partial r^*_k}n_i + k\frac{\partial^2}{\partial r_k^{*2}}n_i + \delta\frac{B_i B_k}{B^2}n_k \tag{15}$$

where only three parameters (plus the ratio of the imposed external magnetic field components, which is essentially a geometrical factor) are left: $\gamma = \gamma_2/\gamma_1$, $k = K/\gamma_1 RV$ and $\delta = \Delta\chi RB^2/\mu_0\gamma_1 V$. The first parameter, γ, is related to viscous effects directly influencing the director time evolution, depending upon Leslie viscosities α_2, α_3: notice that a indirect influence comes also from α_4 which enters the analytical expressions of the Newtonian velocities employed in the present approximate treatment, while the remaining viscous coefficients α_5, α_6 do not influence the director dynamics. The second parameter k is proportional to the average elastic constant, usually of the order of magnitude of 10^{-11} N. The last parameter δ defines the influence of the magnetic field on the director motion.

In order to define completely the probe–fluid interaction, we need to establish boundary conditions of the director components on the probe surface. It is interesting to note that in Eq. (4) only the behaviour of the director field on the surface is present, so that the correction to the isotropic friction coefficient is straightforward once a condition of *strong anchoring*, i.e. of fixed director orientation on the probe surface, $[n_i(t) = n_i^0]$, is assumed. Below we shall discuss this limit case, which corresponds essentially to an infinite anchoring energy of the probe surface on the director. More interesting is the case of *weak anchoring*, which corresponds to negligible anchoring energy. In the following we shall mostly concentrate on this case, which can be defined formally by assuming zero normal derivatives $[\partial n_i(t)/\partial \mathbf{m} = 0]$ of the director components on the surface. Finally, we shall limit our investigation to a limited ensemble of geometrical setups, namely spherical probes with the direction of motion, assumed to be along the laboratory frame axis $\mathbf{e}_3$ and a magnetic field either parallel to the axis of motion, e.g. along $\mathbf{e}_3$ or perpendicular, e.g. along $\mathbf{e}_1$. In other words the factor $B_i B_k / B^2$ will be chosen equal to $\delta_{i,3}\delta_{k,3}$ (parallel) or

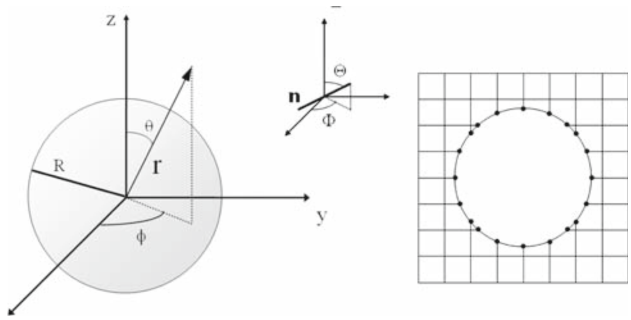

Fig. 1 Spherical probe and grid

Table 1 Leslie coefficient for nematic liquid crystal phases at room temperature

α_i (Pa s)	MBBA	PAA	5CB	E7
α_1	−0.0087	0.0043	−0.006	0.6
α_2	−0.052	−0.0069	−0.07706	−0.13
α_3	−0.002	−0.0002	−0.0042	0.06
α_4	0.058	0.0068	0.0634	0.48
α_5	0.038	0.0047	0.0624	−0.45
α_6	−0.016	−0.0023	−0.0184	−0.26

Data for MBBA, PAA, 5CB are taken from refs. [17–19], respectively

$\delta_{i,1}\delta_{k,1}$ (perpendicular). The numerical solution of Eq. (15) and integration of Eq. (4) can be accomplished by defining a suitable grid of points in space (cfr. Fig. 1) and adopting a standard finite difference scheme to propagate in time the director components n_i, starting from some suitable initial condition. In the following we shall assume that initially the director is everywhere aligned with the imposed magnetic field.

The time evolution of the director components is obtained by using a simple explicit time scheme. Discretisation in space is accomplished by using an almost regular cylindrical grid, i.e. a regular cylindrical grid in the bulk, defined by cylindrical coordinates r_i^*, ϕ_i, z_i or Cartesian coordinates $r_i^* \cos\phi_i, r_i^* \sin\phi_i, z_i$ and an irregular grid for the probe surface, where the director is calculated directly at the points generated by the intersection of the bulk grid and the surface itself. Once the director components are calculated in all grid points at a given time, surface points only are employed to numerically integrate Eq. (4).

From Eq. (4), substituting the expression for the stress tensor of a nematic fluid, one can conveniently write the total friction acting on a spherical probe in rectilinear motion in a nematic fluid in the form $\xi_{//,\perp} = \xi_{\rm iso} c_{//,\perp}$ where $\xi_{\rm iso} = 3\pi\alpha_4 R$ and $c = c_1 + c_2 + c_3 + 1 + c_5 + c_6 + c_{el}$ where the isotropic term is simply obtained by Eq. (6) for $\mu = \alpha_4/2$, and the adimensional factors c_i come directly from the different terms entering the stress tensor σ_{ij}. They are obtained directly from Eq. (4) (see below).

3 Results

We discuss several sets of numerical and asymptotic results. First we present a simplified analysis for the case of a spherical probe in rectilinear motion in the nematic phase of 4-methoxibenzylidene-4′-*n*-butylaniline (MBBA), a well-known liquid crystalline fluid. We assume an average elastic constant $K = 10^{-11}$ N; viscosity coefficients at 25°C are reported in Table 1. Next we discuss a full solution for the case of spherical probes in MBBA. Unless otherwise stated, we assume a unitary value for the ratio between the probe radius and velocity, and a value for the radius fixed to 100 μm, and we study the director dynamics and friction coefficients by varying the scaled number δ. This scaled number defines the influence of the magnetic field on the director motion.

Finally, an analysis of simulations performed in other nematogens, namely PAA (4,4′-dimethoxyazoxy benzene), 5CB (4′-*n*-pentyl-4-cyanobiphenyl) PAA, 5CB and E7 (a mixture of different liquid crystal).

3.1 Axially symmetric case

We start from a simple analytic calculation of the correction to the isotropic friction coefficient valid in the case of an axially symmetric system, by considering i) a spherical probe, ii) a director aligned with the direction of motion, i.e. $n_i = \delta_{i,3}$. This geometrical set-up allows to discuss simplified equations of motion for the director time evolution, and to illustrate anchoring effects. To simplify further the presentation, we neglect in this section elastic and convective terms. We can write director components in the form $n_1 = \cos\Phi\sin\Theta, n_2 = \sin\Phi\sin\Theta, n_3 = \cos\Theta$. If r, ϕ and θ are polar coordinates in space, due to axial symmetry one has simply $\Phi = \phi$ and $\Theta = \Theta(\theta, r, t)$. By combining Eq. (14), one can easily obtain an equation in the unknown function Θ, which describes the time and space dependent angle between the local director and the symmetry axis:

$$\frac{\partial\Theta}{\partial t} = \frac{1}{2}\left(rv_\perp - \frac{{\rm d}v_\parallel}{{\rm d}r}\right)\sin\theta - \frac{\gamma_2}{2\gamma_1}\left[\left(rv_\perp + \frac{{\rm d}v_\parallel}{{\rm d}r}\right)\right.$$
$$\left.\times\sin(\theta - 2\Theta) + r^2\frac{{\rm d}v_\perp}{{\rm d}r}\cos\theta\sin 2(\theta-\Theta)\right]$$
$$-\frac{\Delta\chi B^2}{2\mu_0\gamma_1}\sin 2\Theta \qquad (16)$$

where the components of the Newtonian velocity field are written as $v_1 = v_\perp r_1 r_3$, $v_2 = v_\perp r_2 r_3$, $v_3 = v_\perp r_3^2 + v_\parallel$, and $v_\perp = \frac{3RV}{4r^3}\left(\frac{R^2}{r^2} - 1\right)$, $v_\parallel = V\left[1 - \frac{R}{4r}\left(\frac{R^2}{r^2} + 3\right)\right]$.

In this way it is possible to evidence the angular dependence of the velocity and the director.

On the probe surface, the scaled form of Eq. (15) assumes a particularly simple form:

$$\frac{\partial\Theta}{\partial t^*} = \frac{3}{4}\{\sin\theta + \gamma[\sin(\theta - 2\Theta) - \cos\theta\sin 2(\theta-\Theta)]\} - \frac{1}{2}\delta\sin 2\Theta. \tag{17}$$

Calculation of corrective terms c_i is now relatively easy, and we report here the complete expressions. From Eq. (4) one gets

$$\begin{aligned}
c_1 &= \frac{\alpha_1}{\alpha_4}\int_0^\pi d\theta \sin\theta\cos\Theta\cos^2(\theta-\Theta) \times[\cos\theta\cos(\theta-\Theta)-\cos\Theta]\\
c_2 &= \frac{\alpha_2}{\alpha_4}\int_0^\pi d\theta \sin\theta\cos\Theta\sin(\theta-\Theta)\left(-\frac{1}{2}\sin\theta+\frac{2}{3}\frac{\partial\Theta}{\partial t}\right)\\
c_3 &= \frac{\alpha_3}{\alpha_4}\int_0^\pi d\theta \sin\theta\sin\Theta\cos(\theta-\Theta)\left(\frac{1}{2}\sin\theta-\frac{1}{3}\frac{\partial\Theta}{\partial t}\right)\\
c_5 &= \frac{\alpha_5}{\alpha_4}\int_0^\pi d\theta \sin\theta\cos\Theta\left[\frac{1}{4}\sin 2\theta\sin\Theta+\cos^2\theta\cos\Theta -\frac{1}{2}\cos\Theta\left(1+\cos^2\theta\right)\right]\\
c_6 &= \frac{\alpha_6}{\alpha_4}\int_0^\pi d\theta \sin\theta\cos(\theta-\Theta) \times\left[\sin\Theta\sin\theta\left(\cos^2\theta-\frac{1}{2}\right)+\cos\Theta\cos\theta\left(\cos^2\theta-1\right)\right].
\end{aligned} \tag{18}$$

We can further simplify our result assuming a fixed director orientation on the probe surface. This case can also be considered as the limit behaviour of a spherical probe moving along the director axis of a nematic sample perfectly aligned (infinite magnetic field or $\delta \to \infty$). The adimensional Stokes factor c is obtained by putting $\Theta = 0$ (perfect alignment) in the integral expressions of c_i; one gets:

$$\alpha_4 c = \frac{4}{15}\alpha_1 + \frac{2}{3}\alpha_2 + 1 + \frac{2}{3}\alpha_5 + \frac{4}{15}\alpha_6. \tag{19}$$

In Fig. 2 we show the dependence of the adimensional coefficient as obtained from Eq. (17) and Eq. (19). Notice that the friction is significantly smaller (25%) that in the isotropic case and that only for value of $\delta > 2$, typical of magnetic fields larger that 3 Tesla for the geometry considered here, the asymptotic expression Eq. (19) is acceptable.

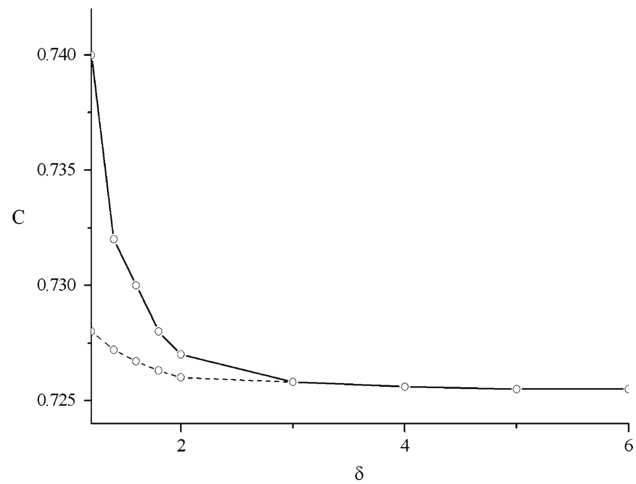

Fig. 2 Adimensional Stokes coefficients versus δ parameter, for the case of a spherical probe in MBBA, calculated according to Eq. (17) for weak anchoring (*full line*) and compared to the asymptotic value obtained from Eq. (19) (*dashed line*)

3.2 Spherical probe in MBBA

Let us now analyse the director behaviour and friction coefficients for the case of a moving spherical probe in MBBA, predicted by the full solution of Eq. (15). We start by discussing the director behaviour in time and space.

In Fig. 3 the time evolution of the director field is represented for the case of the rectilinear motion of a spherical particle in MBBA along the external magnetic field (a) and perpendicular to the external magnetic field (b), for a value of $\delta = 0.2$. Here we adopt a continuous grey scale to represent the director angle with axis of motion $\mathbf{e}_3$, in the plane $\mathbf{e}_1\mathbf{e}_3$. Notice that in the perpendicular case the system is not axially symmetric, so that a slight, but weak dependence of the director patterns would be observed by changing the plane of section. The scale goes from dark grey (director aligned with the axis of motion) to light grey (director perpendicular to the axis of motion). Each snapshot shows the director pattern at a given scaled time: the probe is fixed in space and the fluid is moving from bottom to top. The initial state is chosen to be of perfect alignment with the field in both cases. The first observation which can be made is that the disturbance in the director field is larger in the perpendicular case, while in the parallel case is limited to a close lateral area surrounding the probe. Several factors are influencing the director time evolution: for instance the magnetic torque tends to minimize the director motion, since the initial configuration is fully aligned (minimal magnetic free energy), while the fluid motion acts in a more complex way, although basically it favours alignment of the director along the direction of motion. A stationary state is reached at slightly later times in the perpendicular case, thus causing a longer transition regime corresponding to a time dependent friction coefficient.

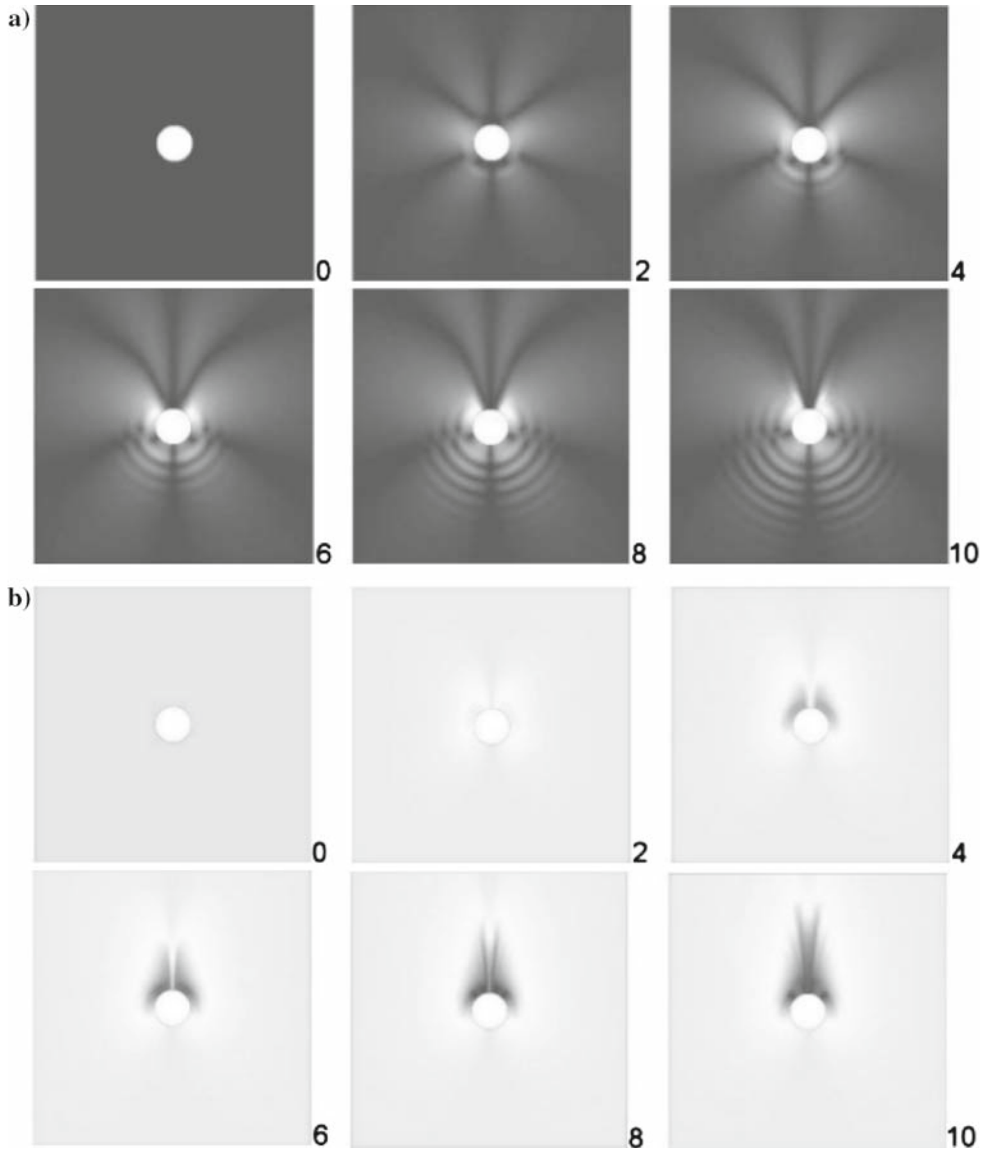

Fig. 3 Time evolution of director field, for a spherical probe in MBBA, $R/V = 1$, $R = 100\ \mu$m and $\delta = 0.2$: direction of motion parallel to field (**a**) and perpendicular (**b**)

We can consider the calculation of the stationary adimensional friction factor c in different liquid crystals, or more exactly in nematic liquid crystalline fluids described by different sets of viscous numbers α_i, reported in Table 1. For simplicity, we shall choose the same geometrical set-ups described in the previous section, namely a sphere of radius of 100 μm, unitary ratio between radius and velocity and $\delta = 0.2$. We also fix arbitrarily $K = 1 \times 10^{-11}$ N for all cases considered. We summarize our findings in Fig. 4. In all cases the friction factor is smaller than one, meaning that the overall friction correction is negative, i.e. the anisotropy of the nematic phase favours the probe motion. This effect is emphasise when the direction of motion is parallel to the direction of the orienting magnetic field, whereas the correction is less important for perpendicular magnetic field. In general, the correction to the isotropic friction is not larger than 30% within the notable exception of E7, which is, however, characterized by relatively high values of Leslie's viscosities (cfr. Table 1).

4 Summary

The purpose of this work was to analyse the dynamical behaviour of a low viscosity nematic liquid crystals in the presence of a micro-size spherical probe in rectilinear motion, by solving numerically the Leslie–Ericksen equations, within clearly stated approximation valid in the case of micrometric objects immersed in a continuous anisotropic medium. To this aim, a very simple computational scheme was employed based on a non-uniform spatial discretisation and an explicit propagation in time. Results were presented for the director

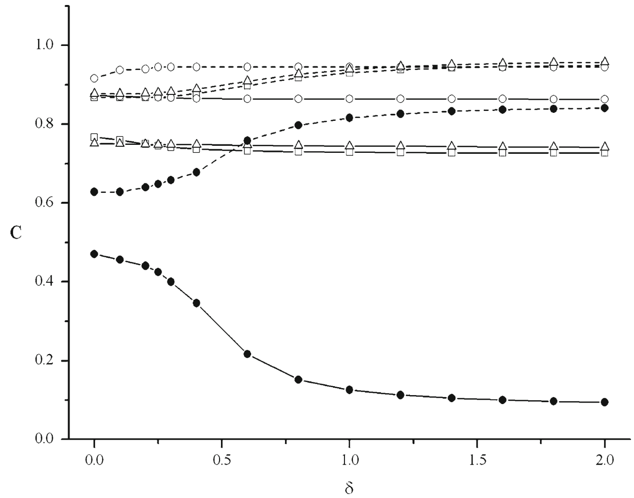

Fig. 4 Stationary adimensional friction factor c, in MBBA (*squares*), PAA (*open circles*), 5CB (*triangles*) and E7 (*filled circles*) for δ=0.2 for a sphere; *full lines* for cases of parallel magnetic field, *dashed lines* for the cases of perpendicular magnetic field

dynamics surrounding the moving probe and the dependence of translational diffusion upon liquid crystal viscoelastic parameters. The time evolution of director field was studied in the presence of an orienting magnetic field in two characteristic situations, i.e. direction of motion parallel and perpendicular to field. Finally a preliminary analysis was given for the case of a spherical probe in rectilinear motion in nematic MBBA (4-methoxibenzylidene-4′-*n*-butylaniline), PAA (4,4′-dimethoxyazoxy benzene), 5CB (4′-n-pentyl-4-cyanobiphenyl) and E7.

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